Problem 1.14

Calculate the mass of a mole of dry air, which is a mixture of N_2 (78% by volume), O_2 (21%), and argon (1%).

[TYPO: Replace "volume" with "pressure." The gaseous components all occupy the same volume.]

Solution

The Hard Way

The mass of a mole of dry air is the sum of the masses of dry air's components, assumed to be nitrogen (N_2), oxygen (O_2), argon (Ar), and carbon dioxide (CO_2).

$$m = \sum_i m_i = m_{N_2} + m_{O_2} + m_{Ar} + m_{CO_2}$$

Mass is obtained by multiplying the number of moles by the molar mass.

$$m = \mathscr{M}_{N_2} n_{N_2} + \mathscr{M}_{O_2} n_{O_2} + \mathscr{M}_{Ar} n_{Ar} + \mathscr{M}_{CO_2} n_{CO_2} \tag{1}$$

Assume that the dry air is an ideal gas mixture so that Dalton's law of partial pressures applies: The total pressure of a mixture of gases is the sum of the pressures that each gas would exert if it were present alone.

$$P = P_{N_2} + P_{O_2} + P_{Ar} + P_{CO_2}$$

As a result, the ideal gas law applied to the mixture

$$PV = n_{\text{total}} RT \quad \rightarrow \quad n_{\text{total}} = \frac{PV}{RT} \tag{2}$$

becomes

$$(P_{N_2} + P_{O_2} + P_{Ar} + P_{CO_2})V = (n_{N_2} + n_{O_2} + n_{Ar} + n_{CO_2})RT$$

$$P_{N_2}V + P_{O_2}V + P_{Ar}V + P_{CO_2}V = n_{N_2}RT + n_{O_2}RT + n_{Ar}RT + n_{CO_2}RT,$$

which implies

$$\begin{cases} P_{N_2}V = n_{N_2}RT \\ P_{O_2}V = n_{O_2}RT \\ P_{Ar}V = n_{Ar}RT \\ P_{CO_2}V = n_{CO_2}RT \end{cases} \quad \rightarrow \quad \begin{cases} n_{N_2} = \dfrac{P_{N_2}V}{RT} \\ n_{O_2} = \dfrac{P_{O_2}V}{RT} \\ n_{Ar} = \dfrac{P_{Ar}V}{RT} \\ n_{CO_2} = \dfrac{P_{CO_2}V}{RT} \end{cases}.$$

Plug these results into equation (1).

$$m = \mathscr{M}_{N_2}\left(\frac{P_{N_2}V}{RT}\right) + \mathscr{M}_{O_2}\left(\frac{P_{O_2}V}{RT}\right) + \mathscr{M}_{Ar}\left(\frac{P_{Ar}V}{RT}\right) + \mathscr{M}_{CO_2}\left(\frac{P_{CO_2}V}{RT}\right)$$

Divide both sides by n_{total}. Since there's just a mole of dry air, $n_{\text{total}} = 1$ mol.

$$\frac{m}{n_{\text{total}}} = \frac{1}{n_{\text{total}}}\left[\mathscr{M}_{\text{N}_2}\left(\frac{P_{\text{N}_2}V}{RT}\right) + \mathscr{M}_{\text{O}_2}\left(\frac{P_{\text{O}_2}V}{RT}\right) + \mathscr{M}_{\text{Ar}}\left(\frac{P_{\text{Ar}}V}{RT}\right) + \mathscr{M}_{\text{CO}_2}\left(\frac{P_{\text{CO}_2}V}{RT}\right)\right]$$

$$\frac{m}{1\text{ mol}} = \frac{RT}{PV}\left[\mathscr{M}_{\text{N}_2}\left(\frac{P_{\text{N}_2}V}{RT}\right) + \mathscr{M}_{\text{O}_2}\left(\frac{P_{\text{O}_2}V}{RT}\right) + \mathscr{M}_{\text{Ar}}\left(\frac{P_{\text{Ar}}V}{RT}\right) + \mathscr{M}_{\text{CO}_2}\left(\frac{P_{\text{CO}_2}V}{RT}\right)\right]$$

$$= \mathscr{M}_{\text{N}_2}\left(\frac{P_{\text{N}_2}}{P}\right) + \mathscr{M}_{\text{O}_2}\left(\frac{P_{\text{O}_2}}{P}\right) + \mathscr{M}_{\text{Ar}}\left(\frac{P_{\text{Ar}}}{P}\right) + \mathscr{M}_{\text{CO}_2}\left(\frac{P_{\text{CO}_2}}{P}\right) \tag{3}$$

Suppose that nitrogen, oxygen, argon, and carbon dioxide contribute 78%, 21%, 0.95%, and 0.05% of the total pressure at sea level, respectively.

$$\begin{cases} P_{\text{N}_2} = 0.78P \\ P_{\text{O}_2} = 0.21P \\ P_{\text{Ar}} = 0.0095P \\ P_{\text{CO}_2} = 0.0005P \end{cases} \quad \rightarrow \quad \begin{cases} \dfrac{P_{\text{N}_2}}{P} = 0.78 \\ \dfrac{P_{\text{O}_2}}{P} = 0.21 \\ \dfrac{P_{\text{Ar}}}{P} = 0.0095 \\ \dfrac{P_{\text{CO}_2}}{P} = 0.0005 \end{cases}$$

Equation (3) then becomes

$$\begin{aligned} \frac{m}{1\text{ mol}} &= \mathscr{M}_{\text{N}_2}(0.78) + \mathscr{M}_{\text{O}_2}(0.21) + \mathscr{M}_{\text{Ar}}(0.0095) + \mathscr{M}_{\text{CO}_2}(0.0005) \\ &= \left(2 \times 14.00674\ \frac{\text{g}}{\text{mol}}\right)(0.78) + \left(2 \times 15.9994\ \frac{\text{g}}{\text{mol}}\right)(0.21) \\ &\qquad + \left(39.948\ \frac{\text{g}}{\text{mol}}\right)(0.0095) + \left[(12.0107 + 2 \times 15.9994)\ \frac{\text{g}}{\text{mol}}\right](0.0005) \\ &\approx 28.97\ \frac{\text{g}}{\text{mol}}. \end{aligned}$$

Therefore, multiplying both sides by 1 mole, the mass of a mole of dry air is

$$m \approx 28.97\text{ g}.$$

The Easy Way

The mass of a mole of dry air is the sum of the masses of dry air's components, assumed to be nitrogen (N_2), oxygen (O_2), argon (Ar), and carbon dioxide (CO_2).

$$m = \sum_i m_i = m_{\text{N}_2} + m_{\text{O}_2} + m_{\text{Ar}} + m_{\text{CO}_2}$$

Mass is obtained by multiplying the number of moles by the molar mass.

$$m = \mathscr{M}_{\text{N}_2} n_{\text{N}_2} + \mathscr{M}_{\text{O}_2} n_{\text{O}_2} + \mathscr{M}_{\text{Ar}} n_{\text{Ar}} + \mathscr{M}_{\text{CO}_2} n_{\text{CO}_2}$$

Divide both sides by n_{total}. Since there's just a mole of dry air, $n_{\text{total}} = 1$ mol.

$$\begin{aligned}
\frac{m}{n_{\text{total}}} &= \frac{1}{n_{\text{total}}}(\mathscr{M}_{\text{N}_2} n_{\text{N}_2} + \mathscr{M}_{\text{O}_2} n_{\text{O}_2} + \mathscr{M}_{\text{Ar}} n_{\text{Ar}} + \mathscr{M}_{\text{CO}_2} n_{\text{CO}_2}) \\
\frac{m}{1 \text{ mol}} &= \mathscr{M}_{\text{N}_2}\left(\frac{n_{\text{N}_2}}{n_{\text{total}}}\right) + \mathscr{M}_{\text{O}_2}\left(\frac{n_{\text{O}_2}}{n_{\text{total}}}\right) + \mathscr{M}_{\text{Ar}}\left(\frac{n_{\text{Ar}}}{n_{\text{total}}}\right) + \mathscr{M}_{\text{CO}_2}\left(\frac{n_{\text{CO}_2}}{n_{\text{total}}}\right) \\
&= \mathscr{M}_{\text{N}_2} x_{\text{N}_2} + \mathscr{M}_{\text{O}_2} x_{\text{O}_2} + \mathscr{M}_{\text{Ar}} x_{\text{Ar}} + \mathscr{M}_{\text{CO}_2} x_{\text{CO}_2}
\end{aligned}$$

Suppose the mole fractions of the components of dry air at sea level are $x_{\text{N}_2} = 0.78$, $x_{\text{O}_2} = 0.21$, $x_{\text{Ar}} = 0.0095$, and $x_{\text{CO}_2} = 0.0005$.

$$\begin{aligned}
\frac{m}{1 \text{ mol}} &= \mathscr{M}_{\text{N}_2}(0.78) + \mathscr{M}_{\text{O}_2}(0.21) + \mathscr{M}_{\text{Ar}}(0.0095) + \mathscr{M}_{\text{CO}_2}(0.0005) \\
&= \left(2 \times 14.00674 \ \frac{\text{g}}{\text{mol}}\right)(0.78) + \left(2 \times 15.9994 \ \frac{\text{g}}{\text{mol}}\right)(0.21) \\
&\qquad + \left(39.948 \ \frac{\text{g}}{\text{mol}}\right)(0.0095) + \left[(12.0107 + 2 \times 15.9994) \ \frac{\text{g}}{\text{mol}}\right](0.0005) \\
&\approx 28.97 \ \frac{\text{g}}{\text{mol}}.
\end{aligned}$$

Therefore, multiplying both sides by 1 mole, the mass of a mole of dry air is

$$m \approx 28.97 \text{ g}.$$